Fractal and multifractal analysis of complex systems

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Many patterns in nature are characterized by a high degree of irregularity and cannot be described by Euclidean geometry. Over the last decades, since the publication of Mandelbrot's famous book "Fractal Geometry of Nature" [1], fractals have produced a significant impact on our understanding of the surrounding world. Statistically self-similar fractal structures are ubiquitous in nature, and fractal concepts provide the base for better understanding of phenomena in diverse scientific fields (such as physics, chemistry, biology, economics, meteorology, etc.).

Simple (mono)fractals characterized by a unique fractal dimension (that measures the degree at which fractal fills the underlying Eucliedian space) are rare, most of natural structures are composed of an infinite set of interwoven fractals (called multifractals), described by a hierarchy of fractal dimensions. Different measures (such as fractal dimension, lacunarity, multifractal spectrum, etc.) have been shown to be useful for analyzing and modeling spatial and temporal data generated by complex systems.

This course aims to provide a basis for understanding and use of fractal and multifractal methods in data analysis and development of corresponding theoretical and computational models. Extensive literature review from various field of science will be presented to illustrate the applicability of these methods.

First, basic concepts related with fractals and multifractals shall be presented, including fractal dimension calculation, multiftactal measures (generalized dimension, singularity spectrum) and lacunarity analysis, with application on temporal and spatial data. Specific methods as Detrended Fluctuation Analysis, Multifractal Detrended Fluctuation Analysis, Detrended Cross Correlation Analysis, Multifractal Detrended Cross Correlation Analysis, that were recently proposed for fractal and multifractal time series analysis will be presented together with examples from the recent literature (fisiological processes, financial temporal series, geophysical signals, climatic data etc.).

Other methods for complexity analysis of temporal series as lacunarity analysis, approximate entropy, sample entropy, cross sample entropy and multiscale entropy will also be presented with applications in medicine, economics and meteorology. For analysis of spatial data methods of calculation of fractal dimension (box-counting, correlation integral) multifractal measures and lacunarity analysis will be presented and illustrated with applications to medical images, geophysical and ecological data.

Development and applications of these tools would not have been possible without substantial progress in computer science. On one hand modern hardware allows acquisition of massive, high quality data, and on the other hand the analyses of such data are often computationally demanding in both terms of speed and memory. Algorithms for Box Counting and the Sand-box method using CUDA on GPGPU platforms will be presented

Bibliography:

[1] B. B. Mandelbrot. The Fractal Geometry of Nature. W. H. Freeman and Company, New York, 1982.

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[3] K. Matia, Y. Aschenazy, H. E. Stanley. Multifractal properties of price fluctuations of stocks and commodities. Europhysics letters 61, 422-428, 2003.

[4] P. Ch. Ivanov, L. A. N. Amaral, A. L. Goldberger, S. Havlin, M. G. Rosenblum, Z. Struzik, H. E. Stanley. Multifractality in human heartbeat dynamics. Nature 399, 461-465, 1999.

[5] J. W. Kantelhardt, E. Koscielny-Bunde, D. Rubski, P. Braun, A. Bunde, S. Havlin. Long-term persistence and multifractality in precipitation and river runoff records. Journal of Geophysical Research 111, DO1106, 2006.

Other publications to be selected considering specific research interests of the audience.